A NUMERICAL PROCEDURE FOR COMPUTING THE VOLTAGE-CURRENT CHARACTERISTICS IN ESP CONFIGURATIONS

E. Lami, F. Mattachini
ENEL, Centro Ricerca Termica e Nucleare
Via A. Pisano, 120
56122 Pisa, Italy

I. Gallimberti, R. Turri
Università di Padova, Dip. Ingegneria Elettrica
Via Gradenigo 6/a
35131 Padova, Italy

U. Tromboni
1999 Informatica Ricerca Sviluppo S.C.r.l.
Via C. Cassan, 34
35121 Padova, Italy

Abstract

A simple and effective numerical procedure is presented for computing the steady-state corona current distribution in electrostatic precipitator configurations. Based on a Finite Difference scheme, it solves the coupled system of the Poisson's equation and the space-charge drift formula[1] in complex bidimensional geometries, taking into account the statistical size distribution of the particulate and the corresponding charging process. A rigorous approach has been used for the space-charge drift which is not based on the commonly used Deutsch approximation.

This procedure represents a valuable design tool for predicting and comparing the performance, in terms of current and electric field distributions, of different electrostatic precipitator configurations and for optimizing their geometric parameters (wire cross-section, wire-wire and wire-plate distances, shape of collecting electrodes, etc.). Examples of application on practical ESP geometries are reported.

1. Introduction

The electrostatic precipitators are the most widely used systems for removing solid particulate from combustion flue gases of thermal power plants. The basic principle underlaying the solid pollutant removal process is to charge solid particulate by means of corona generated ions which then move towards the collecting plate under the effect of the electric field.

In ESP configurations the electric field is generated by high DC or pulsed voltages applied to the emitting electrodes, normally wires, while the collecting plates are earthed. The evaluation of electric field and current density distributions in the interelectrodic space is not a trivial task, maily because of the presence of complex ionization phenomena associated with the corona discharges.

It is possible to calculate ionic charge distribution analytically only in particularly simple electrode geometries and if several simplifying hypotheses are adopted. Several simplified numerical methods have been proposed in the past to solve the problem using either finite difference, charge simulation or finite elements approaches.

In this work a new resolution scheme using a finite difference method has been developed to compute the steady-state corona current distribution in diverging fields, which is not based on the Deutsch approximation and adopts a rigorous approach for the space-charge drift. The procedure has been implemented on a Poisson solver computer package and is being used as a design tool for estimating the electric performance of ESP ducts.

2. Mathematical model

DC energized ESP configurations are characterised by low current electrical coronas, where the ionization processes are confined to very small volumes near the emitting wires, "ionization region", while most of the discharge gap is filled with ions and charged particles drifting in low electric fields, "drift region".

The particulate at the ESP inlet is characterized by a specific granulometric distribution, which can be divided in k classes : if N is the total number density of the particulate and N_i is the number density of the i-th class at any point P inside the precipitator volume, it results :

$$N = \sum_{i=1}^{k} N_i \qquad (1)$$

It is assumed that each class has a specific mobility μ_i and a charge which corresponds to the saturation value given by :

$$q_{si} = 12 \frac{\varepsilon_r}{\varepsilon_r + 2} \varepsilon_o \pi a_i^2 E \qquad (2)$$

where E is the magnitude of the local field, ε_o the dielectric constant, a_i and ε_r the particle radius and the relative dielectric constant respectively.
The space charge density at any point P can be therefore expressed as :

$$\rho = \sum_{i=0}^{k} \rho_i \qquad (3)$$

with :

$\rho_i = N_i\, q_{si};\qquad \rho_o = N_o\, e$

where N_o is the ion number density and e the electronic charge.

The space-charge flow may then be considered to be given by unipolar ions and charged particles drifting with (almost) constant mobility in combined space-charge and externally generated electric fields. If diffusion phenomena are neglected, the equations that describe the ion and particle motion in the "drift region" are those defining the Poisson's field, the current continuity, the conduction current density and the electrostatic potential :

$$\nabla\cdot \mathbf{E} = \sum_{i=0}^{k} \frac{\rho_i}{\varepsilon_o}$$

$$\nabla\cdot \mathbf{J} = 0 \tag{4}$$

$$\mathbf{J} = \sum_{i=0}^{k} \rho_i \mu_i \mathbf{E}$$

$$\mathbf{E} = -\nabla\phi$$

where $\mathbf{E}$ is the electric field, ϕ the electric potential and $\mathbf{J}$ the current density. By combining equations (1) to (4) it is possible to obtain :

$$\rho = N_o e\left[1 + \sum_{i=1}^{k} \frac{N_i}{N_o}\frac{q_{si}}{e}\right]$$

$$\mathbf{J} = \mu_o N_o e\left[1 + \sum_{i=1}^{k} \frac{N_i}{N_o}\frac{\mu_i}{\mu_o}\frac{q_{si}}{e}\right]\mathbf{E} \tag{5}$$

This allows to refer in equation system (4) to the total charge density ρ and to its equivalent mobility :

$$\mu_{eq} = \mu_o \frac{\left[1 + \sum_{i=1}^{k} \frac{N_i}{N_o}\frac{\mu_i}{\mu_o}\frac{q_{si}}{e}\right]}{\left[1 + \sum_{i=1}^{k} \frac{N_i}{N_o}\frac{q_{si}}{e}\right]} \tag{6}$$

which can be calculated at any point P, if the particulate characteristics are known.

It can be shown that the rate of change of the charge density ρ along any field lines is given by[1,2] :

$$\left(\frac{d\rho}{dt}\right)_{\text{field line}} = \frac{\delta\rho}{\delta t} + \mu_{eq}\,\mathbf{E}\,\nabla\rho \tag{7}$$

Applying the Poisson's and the current continuity equations under stationary conditions, with simple manipulations eq (7) may be rearranged as :

$$(d\rho)_{\text{field line}} = -\frac{\rho^2}{\varepsilon_o E}\,dx \tag{8}$$

where x represents a curvilinear coordinate along the field line. Eq (8) can be integrated directly between position x_o and position x along the field line to give :

$$\frac{1}{\rho(x)} - \frac{1}{\rho(x_o)} = \frac{1}{\varepsilon_o}\int_{x_o}^{x}\frac{dx'}{E(x')} \tag{9}$$

also known as the "unipolar charge drift formula": this formula describes exactly, apart from diffusion effects, the charge density variation along its path, $\rho(x)$, as a simple function of position given the initial density $\rho(x_o)$, and independently of the local mobility value.

Application of eq (9) however, requires the exact knowledge of the field lines, which depend on the space charge distribution itself; the solution of the original equation system (4) may be therefore reduced to the coupled solution of the drift formula and the Poisson's equations :

$$\rho(x) = \frac{\varepsilon_o\rho_o}{\varepsilon_o + \rho_o\displaystyle\int_0^x \frac{dx}{E(x)}}$$

$$\nabla^2\phi = -\frac{\rho}{\varepsilon_o} \tag{10}$$

$$\mathbf{E} = -\nabla\phi$$

The necessary boundary conditions are the applied potential ϕ_o and the charge density ρ_o at the border between "drift region" and "ionization region"; if the latter is assumed to be very thin, the boundary conditions can be applied to the emitting wire surface.

3. Finite difference formulation

As stated above, the physical property described by eq (10), derived from the unipolar charge drift formula, is a global property applicable along a field line. In the present work a finite difference scheme has been developed in order to apply such property on any node of an irregular mesh, used by a finite difference Poisson solver[3] to compute the electric field distribution in the discharge gap. The procedure consists of a progressive calculation, starting from points where the density ρ_o is known from the boundary conditions and expanding over all mesh points. Although originally developed for ESP configurations, it has general validity and may be applied on electrode geometries of any complexity.

The finite difference formulation may be briefly described with the aid of fig. 1, which shows a generic mesh node (i,j) together with its 8 neighbouring nodes.

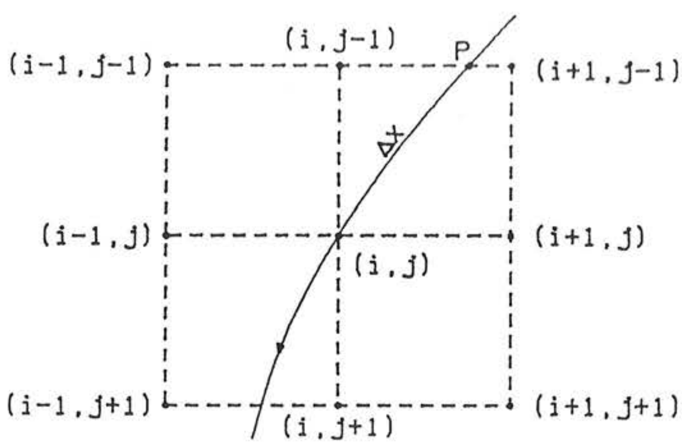


Fig. 1. Finite difference mesh element showing node identifiers and field line.

The task is to determine the charge density $\rho(i,j)$ on node (i,j), knowing the electric field at all the neighbouring nodes and the charge densities on the 3 nodes of order (j-1).
If the point P represents the intercept between the field line passing through node (i,j) and the mesh line of order j-1, the charge density $\rho(P)$ and electric field $E(P)$ may easily be determined, as a first approximation, by linear interpolation of the corresponding values in nodes (i,j-1) and (i+1,j-1).

Denoting Δx the length of the field line between P and node (i,j), eq. (10) may be applied on Δx obtaining :

$$\rho(i,j) = \frac{\varepsilon_o \, \rho(P)}{\varepsilon_o + \dfrac{2\rho(P)\,\Delta x}{E(i,j) + E(P)}} \tag{11}$$

which enables to compute the charge density on a generic node as a function of local variables only.

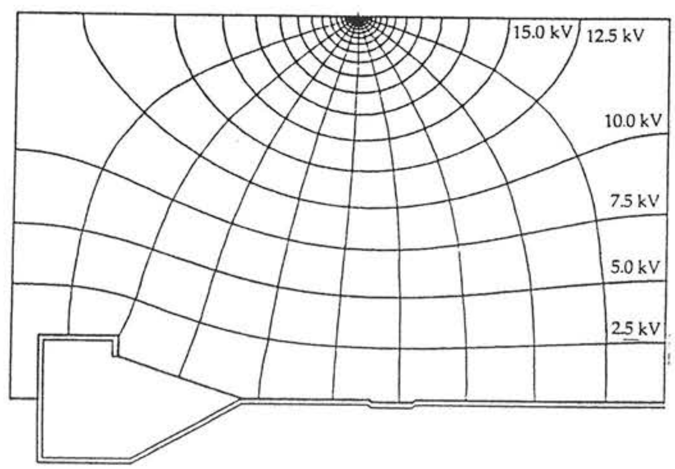
Equation (14) may be applied to determine $\rho(i,j)$ only provided that the charge density on the correct pair of adiacent nodes has been already computed. This has been accomplished by employing a suitable procedure that performs such computation sequentially on all the mesh nodes ordered on the basis of flux direction.

Further, since the electric field is a function of the space charge, the full solution of this problem requires the solution of the non-linear system of coupled equations (10) which can only be obtained with iterative procedures.
In the algorithm adopted the unipolar ion drift formula and the Poisson's equation are solved iteratively until convergence is reached.
The convergence criterium includes the condition that the current at the collecting plates equals the injected current at the corona wires.

4. Simulation results

The procedure described above allows to compute the steady-state corona charge distribution in ESP configurations given the corona current injected into the discharge gap from the ionization region.

(a)



(b)

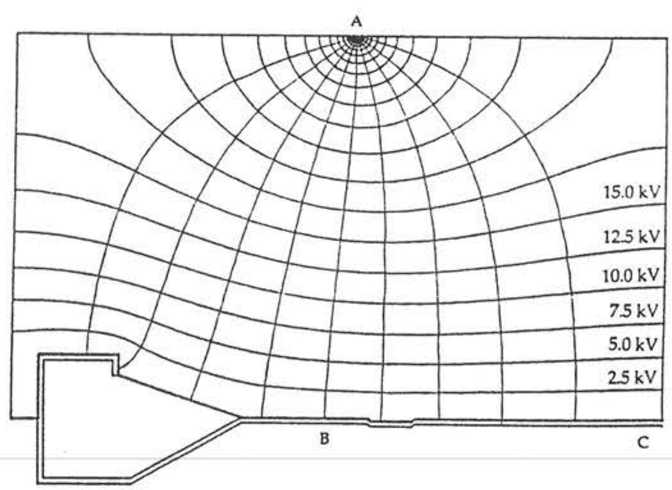


Fig. 2. Equipotential and field lines in a typical ESP configuration.
(a) Laplacian field; (b) Poisson's field.

A computer program has been developed and integrated into a workstation based software package for the solution of the Poisson's equation, whose graphic post-processor enables to perform detailed analysis of voltage, electric field, charge and current density distributions in the discharge gap.

As an example, figs. 2a and 2b show the equipotential and field lines, respectively for the Laplace and Poisson solutions (i.e. without and with a DC negative corona). As expected, the presence of the space-charge modifies both potential and electric field distributions; in particular, the equipotential lines become more dense in the low field region, which corresponds to an increase of the electric field near the plates and a decrease near the h.v. wires. Further, due to space-charge distortion, the electric field lines tend to be more concentrated towards the centre of the collector plate, this effect becoming more marked as the corona current increases. This indicates that the usually accepted Deutsch assumption (the field lines are not modified by the presence of the space charge) can lead to significant errors in practical configuration[5].

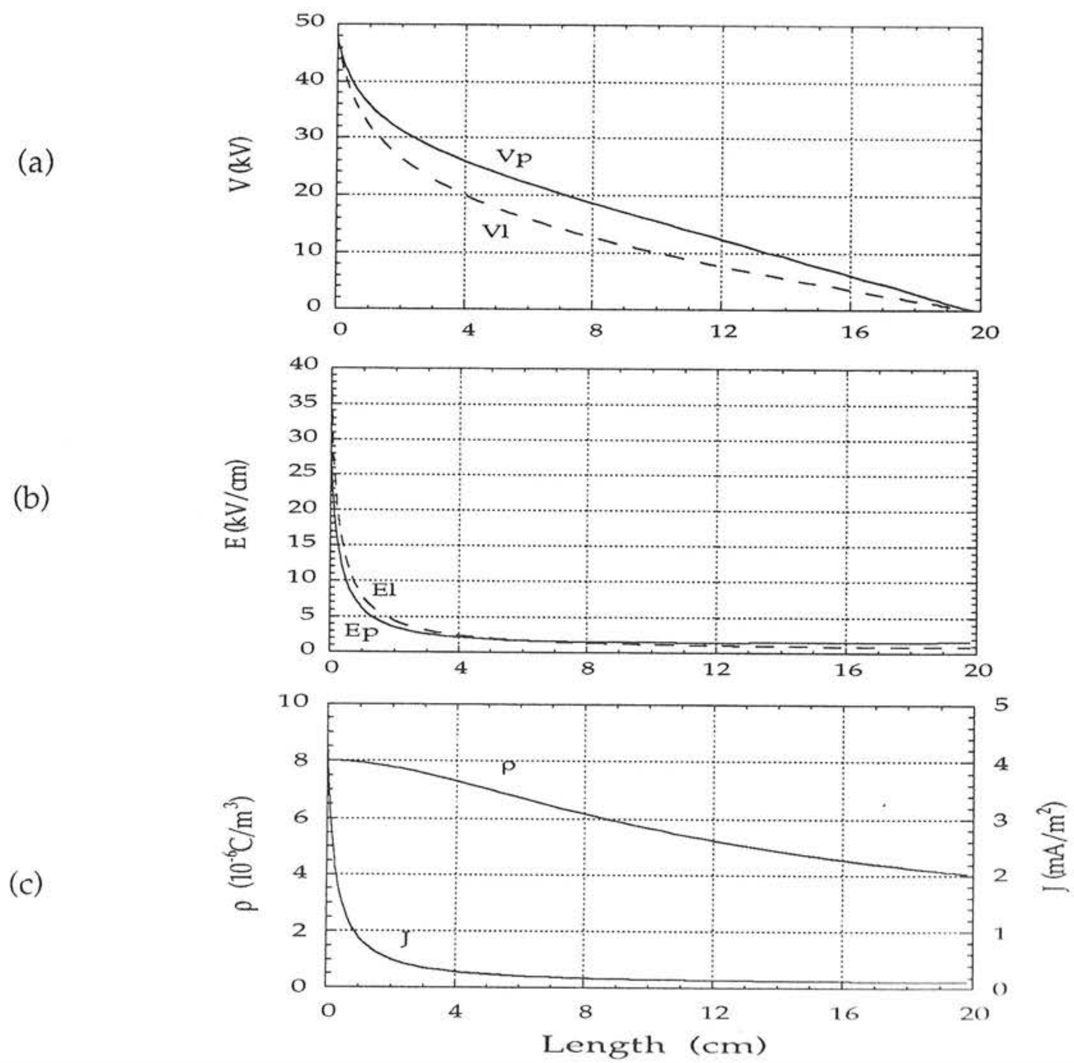


Fig. 3. Potential (a) and electric field (b) distributions along line A-B of fig. 2b. V_p, E_p corresponds to Poisson solution; V_l, E_l corresponds to Laplace solution. (c) Charge and current density distributions along the same line.

The potential and field distributions along the maximum stress line (line A-B in fig. 2b) are reported on fig. 3 (a and b), for both the Laplace and Poisson solutions; fig. 3c shows the corresponding charge and current densities distributions.

In fig. 4 the field, charge density and current density distributions on the collector plate (line B-C in fig. 2b) are shown. The charge density results almost constant over most of the surface, while the current density is greately affected by the surface field and thus increases near sharp corners. The electric field analysis is useful at design stage in order to optimize the collector shape to adjust the current density surface distribution.

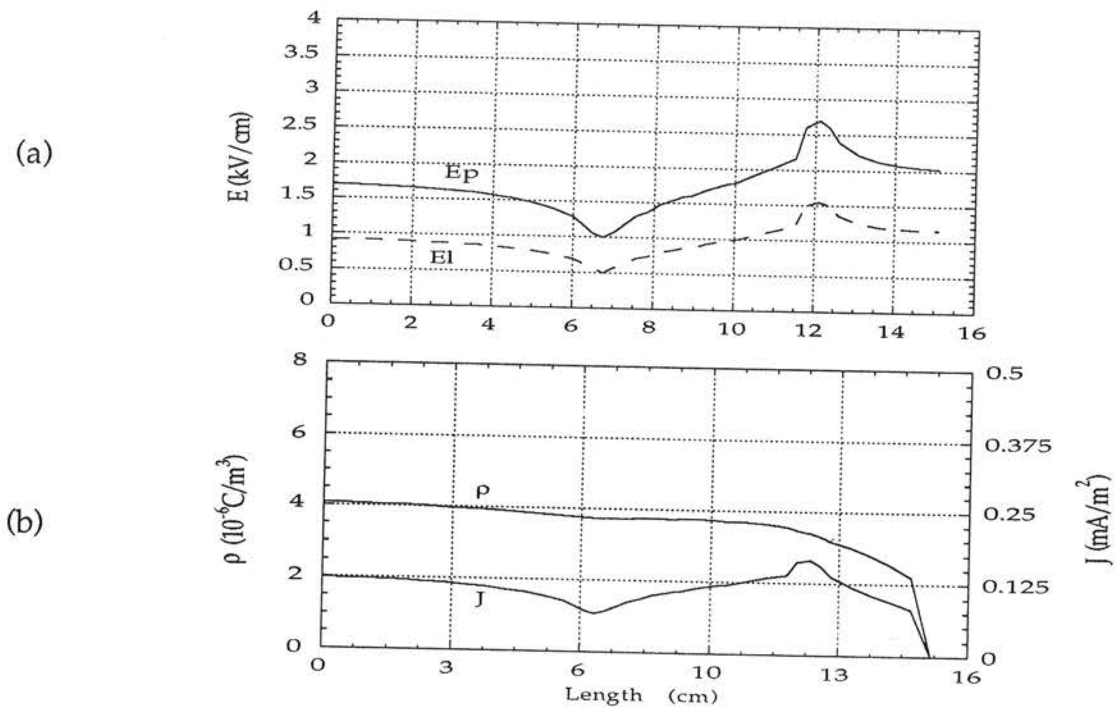


Fig. 4. Electric field (a), charge and current density (b) distribution on the collector plate (line B-C of fig. 2b). The broken line indicates the Laplacian field E_l.

5. Voltage-current characteristics

The proposed model describes on a physical basis the "drift region". In order to compute realistic voltage-current characteristics, it would be necessary to estimate with a good approximation the corona current injected from the "ionization region" as a function of the applied voltage. In such case a comprensive negative corona model, which accounts for its complex structure and for the various phenomena involved at different voltage levels (glow corona, Trichel pulses, breakdown streamers, etc.) should be employed.

Most of the models proposed in the literature are based on the Deutsch assumption, although recently some critical studies have shown a shortcomings of such approximation, in particular when interacting corona sources are involved[4,5]. Given the complexity of the phenomena occurring within the ionization region, it seems unfeasible that models based solely on "critical field" criteria can provide realistic estimates of the corona current injected into the "drift region".

However, in order to compare the current transport capabilities of different geometric configurations in ESP design, simplified corona models and inception criteria can be assumed : in this work the critical field concept has been applied in order to estimate the "maximum corona current" which can be carried in a given geometric configuration. This has been accomplished by gradually increasing the charge density ρ_o on the surface of the emitting wire until the local Poissonian electric field becomes equal to the "onset field" as depicted on fig. 5. Increasing the density ρ_o further would mean to lower the electric field below its limit value and the corona discharge could not phisically exist.

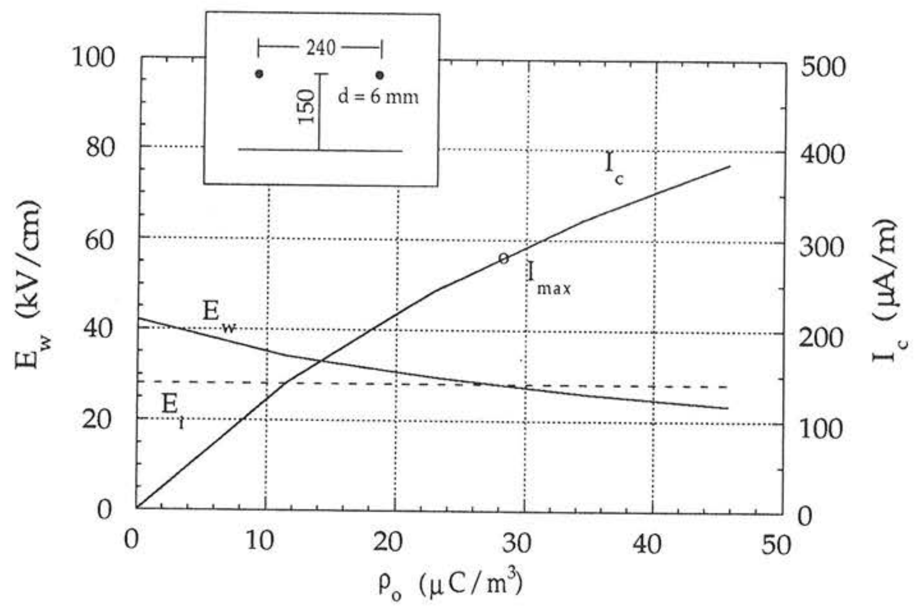


Fig. 5. Example of determination of maximum corona current.
I_c = corona current
E_w = average electric field at wire surface
E_i = onset field.

In corrispondence of this "critical" condition, the "maximum corona current" can be obtained by integration of the current density on the electrode surface. The onset field values have been deduced from current literature values[9,10], as functions of wire radius and gas pressure and temperature.

By applying this procedure for different voltage levels, the "maximum" V-I characteristics can thus be determined; of course the current should be intended as the maximum theoretical value which could be produced by a DC negative corona, and not as a realistic estimate of the experimental corona current.

An example of the V-I_{max} characteristics is reported on fig. 6, where the computed results are compared with experimental measurements[6]. As can be seen, the predicted onset voltage is very close to the measured value, and the maximum current, even if consistently higher, has a similar slope as the measured one.

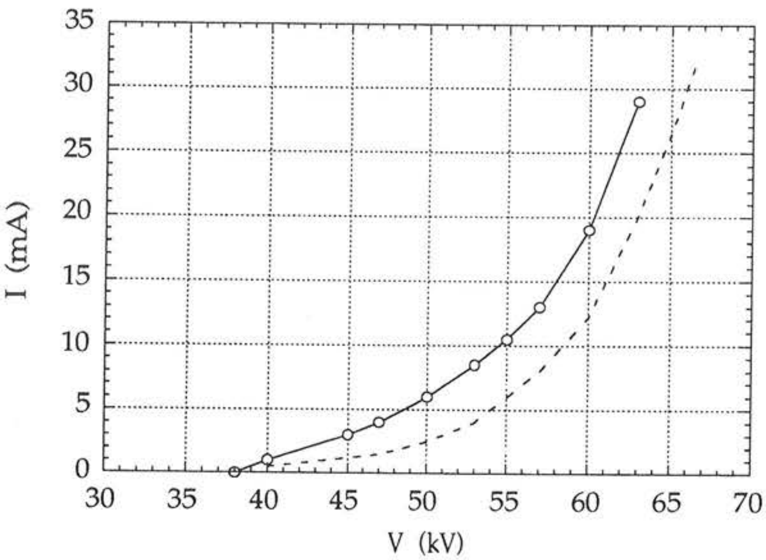


Fig. 6. Comparison of theoretical maximum corona current (-o-) and experimental measurements (---) as a function of applied voltage.

The model does not account for the time-variation of the space-charge, nor for other transient electrical phenomena, which are known to occur in practice, like Trichel pulses and back-corona. Such phenomena can only be accounted for by using a complete time-dependent physical model, such that descibed in a companion paper[8]. However, the model is being used as a design tool for optmizing ESP geometries as regards wire-wire and wire-plate distances, wire diameters and working voltages.

An example of application is given in figs. 7 and 8, where, given the wire diameter (6 mm) and duct size (300 mm width and 3 m length), the optimum wire separation, and consequently the number of wires per duct, is determined.

In fig. 7 the V-I_{max} characteristics are shown for various wire separation distance. As expected, I_{max} increases with wire separation, due to the reduced interactions of the corona space charge between adjacent wires. However, since the duct length is fixed, the larger the wire separation the less is the number of wires which can be fitted.

The maximum total corona current, determined by multiplying I_{max} for the total emitting wire length, is shown on fig. 8 as a function of wire separation for various voltage levels. It clearly appears that there is a non-linear relationship between maximum total current and wire separation, with a maximum around 250-300 mm which is taken, for this specific example, as optimum distance.

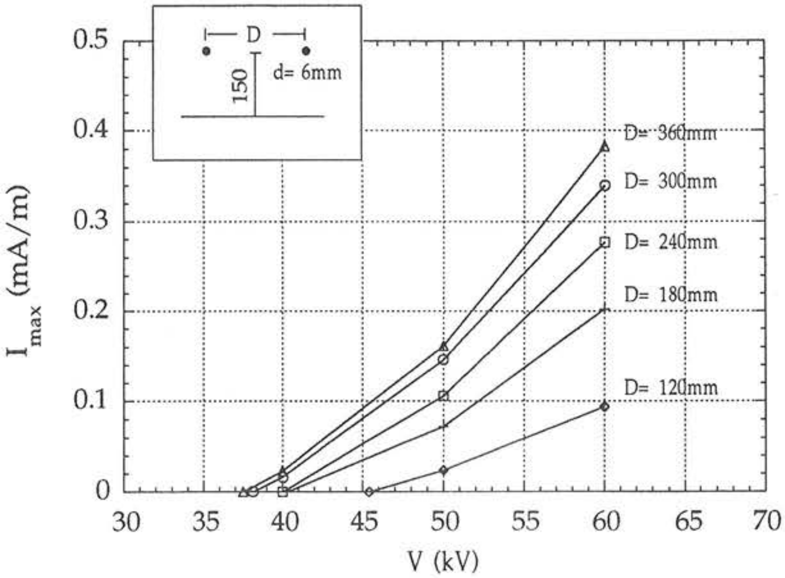


Fig. 7. V-I_{max} characteristics for different wire separation distances.

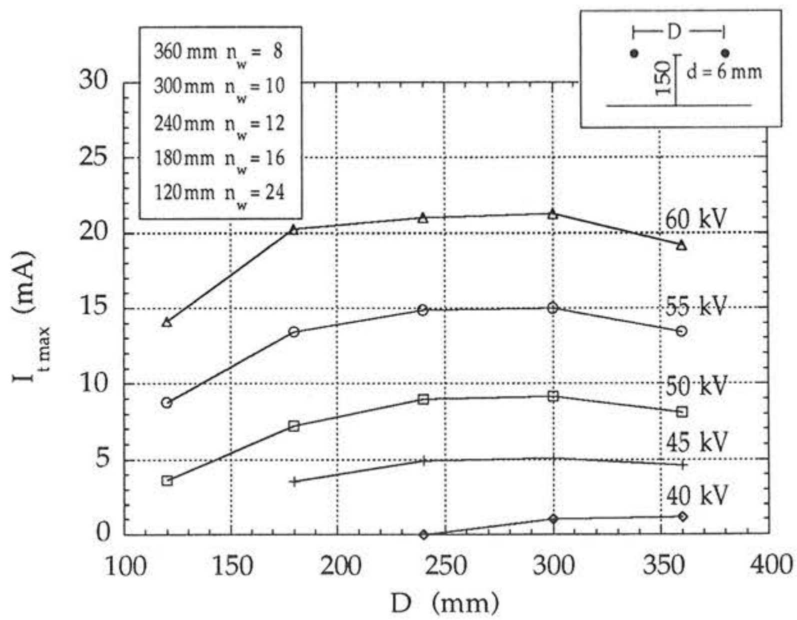


Fig. 8. Maximum total current as a function of wire separation for different voltage levels.

6. Conclusions

In this work a numerical procedure is described for computing steady-state DC corona regimes in ESP configurations.

By applying the Poisson's and current continuity equations, the charge drift formula has been rearranged and expressed as a function of position only, independent of time and mobility. This has enabled to develop a Finite Difference scheme which has general validity and is applicable to electrode geometries of any complexity.

The procedure has been used to determine the space charge and current density distributions knowing the applied potential and the initial current density at the emitting wires, which can be either measured experimentally or determined using a physical model of the ionization region.

However, in order to assess the current transport characteristics of a given ESP geometry there is no need to model precisely the ionization region of the corona discharge but rather to estimate the maximum current which can flow into the gap. An iterative procedure based on the critical field concept has been applied to compute the maximum corona current, thus enabling to determine the voltage-current characteristics of any desired configuration. The effects of charged particulate distributions are accounted for in the solution of the Poisson equation.

The procedure has been implemented in a computer program presently used by ENEL for design optimization of both conventional and innovative ESP installations.

References

1. J.E. Jones. *"On the drift of gaseous ions"*, Journal of Electrostatic, 27, pp. 283-318, 1992.

2. R.S. Sigmond. *"Simple approximate treatment of unipolar space-charge-dominated coronas : the Warburg law and the saturation current"*, J. Appl. Phys., 53, pp. 891-898, 1982.

3. *"FP-Solver : user manual"*, 1999 Informatica Ricerca Sviluppo, Padova, Italy.

4. A. Bouziane, K. Hidaka, M.C. Tapamacioglu, R. Waters. *"Verification of direct-current corona models employing the Deutsch approximation"*, Gaseous Dielectric VI, Pergamon Press, Knoxville, 1990.

5. J.E. Jones, H. Davies. *"A critique of the Deutsch assumption"*, J. Phys. D. :Appl. Phys, 25, pp 1749-1759, 1992.

6. S. Cristina, G. Dinelli, M. Feliziani. *"Numerical computation of corona space charge and V-I characteristics in DC electrostatic precipitators"*, IEEE Trans. on Industry Applications, vol. 27, pp. 147-153 1991.

7. B. Bellagamba, F. Lami, F. Mattachini, A. Pigini, C. Valagussa, A. Martinelli. *"Laboratory tests for the study of fluid flow and corona characterization inside wire-plates structures of electrostatic precipitators"*, V ICESP, Washington DC, 1993.

8. B. Bellagamba, F. Lami, F. Mattachini, I. Gallimberti, R. Turri, A. Gazzani, U. Tromboni, A. Pasinetti, R. Sala. *"A mathematical model for simulation of large scale electrostatic precipitators"*, V ICESP, Washington DC, 1993.